Analysis of an array of wires in a low-frequency time-harmonic magnetic field

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ABSTRACT

Wired layers are often encountered in the cable industry, as in the armoring of three-core submarine power cables for off-shore wind farms. In this application, an array of steel wires is subjected to the time-harmonic magnetic field generated by balanced three phase currents. As wires are made of both conductive and magnetic material, induced currents and hysteresis must be accurately modelled in order to design optimized and cost-effective connections. Approaching this problem with realistic finite-element models is computationally expensive. We propose here a Method-of-Moments formulation where the wires are treated as 1-D structures.

KEYWORDS

Submarine power cables, Method of Moments, Eddy currents.

INTRODUCTION

In electrical engineering there are many applications in which one has to compute low frequency AC or DC magnetic fields and related quantities in the presence of many magnetic and/or conductive wires. An example can be found in high voltage submarine cables, which are typically used to connect the off-shore hub to other hubs or to the mainland power network. The armour of these kind of AC power cables is composed by many iron wires. The computation of the magnetic field inside and outside the shield is required. Hysteresis and eddy current phenomena are to be taken into account.

When analyzing this problem by the Finite Element Method (FEM), one has to discretize the wires into fine elements in order to consider these effects. Moreover, the FEM requires to discretize not only the magnetic and/or conductive wires, but also the air which surrounds them, and to introduce an artificial boundary to the computational domain. For inherently 3D geometries that cannot be reduced to 2D, this leads to numerical models with a large number of degrees of freedom, whose solution is very demanding in terms of both memory and computation time. As a result, any analysis that requires many simulation runs, such as sensitivity analysis or design optimization, can become unfeasible even using state-of-the-art commercial electromagnetic simulation software.

This paper proposes an alternative approach, based on the Magnetostatic Moment Method [1]. The advantages of this method are that only the magnetic material parts are discretized, and there is no need either to discretize the air domain, or to introduce fictitious computational boundaries and associated boundary conditions.

MAGNETOSTATIC FORMULATION

We consider the equations of magnetostatics [5]:

$$\nabla \times \mathbf{H} = \mathbf{J}_S, \tag{1a}$$

$$\nabla \cdot \mathbf{B} = 0, \tag{1b}$$

where $\mathbf{J}_S$ is a known current distribution. We partition the physical space into two regions: a region Ω_M occupied by a magnetic material with relative permeability μ_r and a region Ω_0 occupied by a material with relative permeability 1, *e.g.* air. The constitutive relation relating $\mathbf{B}$ and $\mathbf{H}$ can be written as

$$\mathbf{B}(\mathbf{x}) = \begin{cases} \mu_0 \mu_r \mathbf{H}(\mathbf{x}), & \mathbf{x} \in \Omega_M, \\ \mu_0 \mathbf{H}(\mathbf{x}), & \mathbf{x} \in \Omega_0. \end{cases} \tag{2}$$

We also consider the magnetization vector

$$\mathbf{M} = \frac{\mathbf{B}}{\mu_0} - \mathbf{H}. \tag{3}$$

Inserting (2) into (3), and defining $\alpha = 1 - \frac{1}{\mu_r}$ it can be seen that

$$\mathbf{M}(\mathbf{x}) = \begin{cases} \frac{\alpha}{\mu_0} \mathbf{B}(\mathbf{x}), & \mathbf{x} \in \Omega_M, \\ \mathbf{0}, & \mathbf{x} \in \Omega_0. \end{cases} \tag{4}$$

Relation (4) shows that $\mathbf{M}$ vanishes outside the magnetic domain Ω_M. Therefore, if we manage to write an equation for the magnetization, only the quantities inside Ω_M need to be considered as unknowns of the problem. This is clearly advantageous for certain geometrical configurations of Ω_M, such as when Ω_M is composed of thin wires.

With this aim in mind, following [6], we now proceed to derive an equation for the magnetization which is valid in the magnetostatic regime. By using (3) inside (1a), the equations of magnetostatics (1a) and (1b) can be rewritten in terms of $\mathbf{B}$ and $\mathbf{M}$

$$\nabla \times \mathbf{B} = \mu_0 [\mathbf{J}_S + \nabla \times \mathbf{M}], \tag{5a}$$

$$\nabla \cdot \mathbf{B} = 0. \tag{5b}$$

It is useful to split the magnetic field $\mathbf{B}$ into two terms

$$\mathbf{B} = \mathbf{B}_S + \mathbf{B}_M. \tag{6}$$

The first term $\mathbf{B}_S$ is directly the result of the current source $\mathbf{J}_S$

$$\nabla \times \mathbf{B}_S = \mu_0 \mathbf{J}_S, \tag{7a}$$

$$\nabla \cdot \mathbf{B}_S = 0. \tag{7b}$$

The second term $\mathbf{B}_M$ is due to magnetization $\mathbf{M}$ induced in the material

$$\nabla \times \mathbf{B}_M = \mu_0 \nabla \times \mathbf{M}, \tag{8a}$$

$$\nabla \cdot \mathbf{B}_M = 0. \tag{8b}$$