REVIEW OF UNDERGROUND CABLE IMPEDANCE AND ADMITTANCE FORMULAS

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ABSTRACT

Cable impedance and admittance formulas are essential to study steady-state and transient phenomena on a cable. Pollaczek derived the earth-return impedance of an underground cable in 1928. Wedepohl and Wilcox derived the cable internal impedance and admittance in 1973. Then, the formulation of the cable impedance and admittance matrices was generalized and implemented into well-known EMTP as a subroutine "Cable Constants" in 1976 by Ametani. Since then a number of transient simulations on cable systems have been carried out using the EMTP, and there are many papers discussing the above mentioned work and deriving new formulas, either accurate or approximate. This paper has reviewed and summarized the previous publications to give an idea of what are the cable impedance, admittance, and the EMTP simulations of the cable transients.

KEYWORDS

Cable; impedance; admittance; transient; EMTP.

I. INTRODUCTION

A number of underground cable transmission systems are under construction and/or are planned in many countries [1-3]. For the insulation design and coordination of an underground cable, it is essential to predict and investigate possible over-voltages. Cable impedance and admittance formulas are necessary to study transient and steady-state phenomena on the cable.
The impedance and admittance formulation of a cable is far more complicated than that of an overhead line, because even a single-phase cable consists of two conductors at least, i.e. a core conductor and a metallic sheath (shield) in the case of a single-core coaxial cable (SC cable) [4, 5]. Also a long high-voltage SC cable is quite often cross-bonded, similar to overhead line transposition. Furthermore, a so-called pipe-type cable (PT cable), such as a POF cable, is composed of three-phase cables installed within a conducting pipe. Then the PT cable becomes a seven-conductor system.
An impedance formula of a cylindrical conductor was derived by Schelkunoff in 1932 [6]. The impedance and admittance formulas of an SC cable were developed by Wedepohl and Wilcox [4]. The impedance and admittance formulas of a PT cable, where an inner conductor is eccentric to the pipe centre, were developed by Brown and Rocamora [7]. The earth-return impedance of an underground cable was derived by Pollaczek in 1926[8]. The formulas have been generalized and implemented into well-known EMTP (Electro-Magnetic Transients Program) as a subroutine "Cable Constants" in 1976 by Ametani in the Bonneville Power Administration, US Department of Energy [5, 9].
This paper summarizes and reviews the impedance and admittance formulation of three-phase SC and PT cables. Also, problems of the formulas and their applications are reviewed, and a recent trend of the cable impedance and admittance calculations is explained.

II. IMPEDANCE AND ADMITTANCE FORMULATION

A.Formulation of impedance and admittance

The Impedance and admittance of a cable system are defined in the two matrix equations [5].

$$d\,\mathbf{V}/dx = -\mathbf{Z}\cdot\mathbf{I} \qquad d\,\mathbf{I}/dx = -\mathbf{Y}\cdot\mathbf{V} \tag{1}$$

where $\mathbf{V}, \mathbf{I}$: voltage and current vectors at distance x,
$\mathbf{Z}, \mathbf{Y}$: square matrices of impedance and admittance.

In general, the impedance and admittance matrices of a cable can be expressed in the following forms [5].

$$\mathbf{Z} = \mathbf{Z}_i + \mathbf{Z}_p + \mathbf{Z}_c + \mathbf{Z}_0 \tag{2}$$

$$\mathbf{Y} = s\cdot\mathbf{P}^{-1},\ \mathbf{P} = \mathbf{P}_i + \mathbf{P}_p + \mathbf{P}_c + \mathbf{P}_0 \tag{3}$$

where $\mathbf{P}$ is a potential coefficient matrix and $s = j\omega$.

In the above equations, the matrices with subscripts "i" concern an SC cable and the matrices with subscript "p" and "c" are related to a pipe enclosure. The matrices with subscript "o" concern cable outer media, i.e. air space and earth. When a cable has no pipe enclosure, there exists no matrix with subscripts "p" and "c".
In the above formulation implemented in the EMTP, the following assumptions are made [5].

- The displacement currents and dielectric losses are negligible.
- Each conducting medium of a cable has constant permeability.
- The pipe thickness is greater than the penetration depth of the pipe wall for the PT cable case.

The details will be explained in the following sections.

B. Impedance matrix

B1. Internal impedance of a single-core coaxial cable (SC cable)

Assume that an SC cable consists of a core, sheath and armor as shown in Fig. 1(a). The impedance matrix is given in the following form.

$$\mathbf{Z_i} = \begin{bmatrix} \mathbf{Z}_{i1} & \dots & \mathbf{0} \\ \dots & \dots & \dots \\ \mathbf{0} & \dots & \mathbf{Z}_{in} \end{bmatrix} \tag{4}$$

All the off-diagonal sub-matrices of $\mathbf{Z}_i$ are zero.

A diagonal sub-matrix $\mathbf{Z}_{ij}$ ($j = 1,\dots,n$ for an n-phase SC cable) expresses the self-impedance matrix of one phase SC cable, which is given by: