A7.3

Thermal design of controlled backfills

ARGAUT, P., LESUR F., SAGEM SA, Montereau, France
MERMET G., PROTAT F., SPORŘEL U., SUPELEC, Gif sur Yvette, France

Résumé

La capacité de transport des câbles de puissance dépend fortement du milieu environnant. Les caractéristiques thermiques du sol sont un paramètre critique qui conditionne le risque de migration d'humidité et d'emballement thermique. Des remblais spéciaux peuvent remplacer avantageusement le sol qui a été creusé.
Le texte ci-après présente des résultats comparatifs à partir de quelques méthodes permettant de calculer la résistance thermique externe T_4 lorsque le sol n'est pas homogène.
Des données économiques peuvent dès lors être utilisées pour un dimensionnement optimisé des remblais, selon le critère du ratio entre puissance transitée et coût global de la tranchée.

Abstract

Ampacity of power cables is significantly dependent on the surrounding medium. The thermal characteristics of the soil is a critical parameter increasing the risk of moisture migration and thermal instability. Special backfills may be used to replace advantageously the local excavated soil.
The following paper presents some comparative results from a few methods to calculate external thermal resistances T_4 when the soil is not homogeneous.
Economical data may be introduced to lead to an optimum design of backfills, according to a criterion of ratio between cable ampacity and global cost of a trench.

1. Heat transfer for buried cable systems

Total losses generated in buried power cables flow through the soil to the ground surface and are dissipated into the atmosphere. The conductor temperature rise of these cables is due for its major part to the soil, and analysis to one factor sensitivity studies show that the depth of laying and thermal characteristics of the soil are critical parameters.

Granular particles greatly affect the thermal behaviour of the soil according to their constituents, size distribution, density and moisture content. Cavities between the more or less compacted particles are filled with water or air. If the moisture migrates from the soil, the thermal resistivity rises, increasing the cable temperature and intensifying the losses. It contributes to speed up moisture migration and a thermal instability arises.

Because thermal resistivities of mineral constituents and water are out of proportion with the one of air, it is essential to maximise the amount of solid and water. A high density due to compaction improves the series of parallel paths in the global structure. Therefore special backfills (selected sands, stabilised or fluidised backfills) may be used to replace the local excavated soil.

2. Cable systems in backfills

The computation of ampacities of cables systems in backfills is a current practice. It applies to cables laid in a well-conducting material to improve heat dissipation and to ducts installed in layers of concrete. In both configurations, the surroundings are a material which has a different thermal resistivity from that of the native soil.

We will consider the installation of three cables in flat formation described in Electra #98 [4].

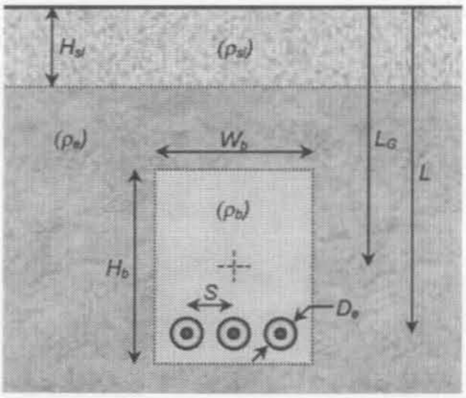

fig.1 : Three cables in a backfill, flat formation

A list of symbols is given at the end of the document.

3. Superficial layer

CIGRE WG02 of SC21 recommends in [4] a method to calculate the effective external thermal resistance of cables laid in a backfill. The heat path between cable and soil surface contains regions having different values of thermal resistivity, when the conventional formulae given in IEC Publication 60287 [2] are based on the assumption of a single value of thermal resistivity throughout the ground. In the present paper, we added the feasibility of taking into account an additional discontinuity of thermal resistivity : a superficial layer of height H_{sl} is considered. It may represent the upper layer of the soil exposed to solar radiation, of higher thermal resistivity in summer when drying out of the soil occurs.

4. Reference example

The well-known example of [4], illustrated by figure 1, is also used for our calculations :

D_e = 100 mm, S = 120 mm,
L = 1450 mm, L_G = 1350 mm,
W_b = 400 mm, H_b = 500 mm.

Original values of thermal resistivity of soil and backfill (respectively 3,0 and 1,0 K.m/W) were judged a little high. We adopted values in common use in France, in accordance with operating conditions of [3] :

ρ_e = 0,85 K.m/W, ρ_b = 0,7 K.m/W.

We introduced a superficial layer of higher thermal resistivity :

H_{sl} = 500 mm, ρ_{sl} = 1,2 K.m/W.

This basic configuration is applied to compare different methods of calculation, dealing with steady-state thermal conditions. The singularity of discrete changes of thermal resistivity is treated as a two dimensional problem in a plane perpendicular to the cable axis.

5. An iterative solution in a transformed plane

The first method involves the numerical solving by successive iterations of the differential equation of heat conduction in a conformal transformed plane [4]. It relies on three main assumptions :

- *hyp.1* : the ground surface is isothermal (Kenelly hypothesis).
- *hyp.2* : the surface of the studied cable is isothermal, that amounts to suppose its thermal conductivity is infinite. A self resistance between the cable and the ground surface can be defined from hypothesis 1 and 2.
- *hyp.3* : when the installation includes several loaded cables, a partial thermal field is calculated for each cable, by supposing that it is the only heat source, all other cables being replaced by soil. Then the partial fields are added and the temperature rise on each cable can be estimated (restricted application of the principle of superposition).

Each cable is in turn the reference cable, considered as a line source of depth y_1. The centre of the other cables and the location of the boundaries between regions of different thermal resistivity (backfill and superficial layer) are mapped in the transformed plane by the function :

$$w = \Phi(z) = \ln\left[\frac{z + j|y_1|}{z - j|y_1|}\right] \quad (1)$$

The conformal transformation simplifies a problem in the original plane (z plane) with infinite dimensions, to a rectangle of finite dimensions (w plane) where the upper boundary represents the ground surface and the lower boundary characterises the cable surface. The differential equation of heat conduction is invariant under such a transform. The temperatures at corresponding locations in the z plane and the w plane are the same when the boundary conditions are the same. In the transformed plane, the isotherms become straight lines parallel to the line representing the ground surface, and the flux lines are represented by a second set of parallel lines perpendicular to the isotherms.

An analogous resistance network covering the rectangular area of the transformed plane is then developed, where nodal points are linked by a grid of discrete elements characterising areas of different thermal resistivities. From an initial linearly spaced distribution of temperatures between the soil and cable surfaces, it is possible to evaluate the temperatures of the mapped nodes with a standard method of numerical analysis or a recurrence equation, allowing an iterative solution.

The distribution of nodal temperatures yields the value of the quantity of heat flowing per time unit from the reference cable to the ground surface. Using in turn each cable as the reference one, the methods determines the complete matrix which elements are the self thermal resistance of the i^{th} cable in the system, or the mutual thermal resistance between cable i and cable k, playing the same role as the external thermal resistance T_4 given in IEC Publication [2].

6. The Neher-McGrath method

The conventional formulae recommended by IEC Publication 60287 [2] to calculate external thermal resistances T_4 is only applicable with the assumption of a single value of thermal resistivity throughout the ground (homogeneous soil). The Neher-McGrath method has been integrated in the international standard to consider ducts embedded in concrete. This approach may obviously be extended to any envelope such a backfill : a correction is added to account for the difference in the thermal resistivities of the envelope and the native soil.

The method is based on two assumptions, added to hyp.1 to 3 in the previous section :

- *hyp.4* : the rectangular backfill is replaced by a circular backfill with the same capacity of heat dissipation.
- *hyp.5* : the surface of the equivalent circular backfill is isothermal. In the transformed plane, the boundary of the backfill is a straight horizontal line.

The equivalent radius of the isothermal circle depends on the rectangular backfill sides :

$$x = \min(H_b, W_b) \quad and \quad y = \max(H_b, W_b) \tag{2}$$

$$r_b = \exp\left[\frac{1}{2}\frac{x}{y}\left(\frac{4}{\pi} - \frac{x}{y}\right)\ln\left(1 + \frac{y^2}{x^2}\right) + \ln\left(\frac{x}{2}\right)\right] \tag{3}$$

With :

$$u_b = \frac{L_G}{r_b} \tag{4}$$

the geometric factor G_b introduced in [6] is :

$$G_b = \ln\left(u_b + \sqrt{u_b^2 - 1}\right) \tag{5}$$

The correction is added algebraically to T_4''', the external resistance of the cable or duct :

$$corr\left(T_4'''\right) = \frac{N}{2\pi}\left(\rho_e - \rho_b\right)G_b \tag{6}$$

For centre cable of the reference example (if cables have approximately equal losses) :

$$u = \frac{2L}{D_e} \tag{7}$$

$$T_{40}''' = \frac{\rho_e}{2\pi}\left[\ln\left(u + \sqrt{u^2 - 1}\right) + \ln\left(1 + \left(\frac{2L}{S}\right)^2\right)\right] \tag{8}$$

$$T_4 = T_{40}'''\frac{\rho_b}{\rho_e} + corr\left(T_4'''\right) = T_{4b} + T_{4s} \tag{9}$$

The second formulation separates clearly the terms relative to the backfill T_{4b}, and the surrounding soil T_{4s}, where :

$$T_{4b} = \left(\frac{T_{40}'''}{\rho_e} - \frac{N}{2\pi}G_b\right)\rho_b \tag{10}$$

$$T_{4s} = \frac{N}{2\pi}\rho_e G_b \tag{11}$$

The equivalent expression is convenient to evaluate the temperature on the backfill boundary (supposed isothermal according to hyp.5), of particular interest to fix a critical temperature when drying-out of the soil occurs.

7. The El-Kady & Horrocks method

The previous method substitutes the original rectangular backfill for a circular shape (equ.3). This approximation is only valid for ratios of *y/x* less than 3. El-Kady and Horrocks [6] proposed a table of extended values of the geometric factor G_b to overcome this restriction, from calculations with a finite-element technique, as a function of backfill depth/height (L_G/H_b) and height/width (H_b/W_b) ratios.

With regard to the Neher-McGrath approach, hyp.5 of an isothermal backfill/soil interface is still assumed, but equations 2 to 5 become obsolete and G_b is given by a table.

8. The «corrected backfill depth» method

Despite of this refinement to the Neher-McGrath approach, the previous method [6] still assumes that the backfill surface is isothermal. But cables are usually laid in the lower part of the backfill, and the heating of the backfill is higher at the bottom than at the top. So we have adopted a new approach which does not formulate the restrictive hyp.5.

Let us consider a single cable in a uniform soil, at a depth *L*, assumed as an equivalent heat line source of co-ordinate y_1. The set of isothermal lines are eccentric circles of radius *r* :

$$\sqrt{L^2 - r^2} = y_1 \tag{12}$$

Especially for the isothermal cable surface :

$$y_1 = \sqrt{L^2 - \frac{D_e^2}{4}} \tag{13}$$

The depth value of the centre of the equivalent circular backfill L_b is deduced from its radius r_b (equ.3).

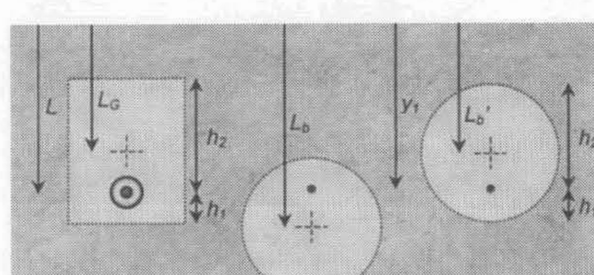

fig.2 : Equivalent circular backfills (original and corrected depths)

We have chosen a corrected depth L_b' so that the heights of backfill material above and below the cable system follow the same ratio as the actual rectangular backfill ($h_2/h_1 = h_2'/h_1'$) :

$$L_b' = y_1 + r_b\left[\frac{L_G - L}{H_b} - \frac{1}{2}\right] \tag{14}$$

Now the new equivalent backfill is established, let us calculate the conformal transform of its boundary :

$$w = u + jv = \Phi\left(-jL_b^{'} - jr_b\cos\alpha + r_b\sin\alpha\right) \quad (15)$$

where : $\alpha \in [0,2\pi]$,

According to equ.1, developing the exponential terms of equ.15, and identifying real and imaginary parts : (16)

$$\begin{cases} e^u \cos v.r_b \sin\alpha + e^u \sin v\left(L_b^{'} + r_b\cos\alpha + |y_1|\right) = r_b \sin\alpha \\ e^u \sin v.r_b \sin\alpha - e^u \cos v\left(L_b^{'} + r_b\cos\alpha + |y_1|\right) \\ \qquad = -\left(L_b^{'} + r_b\cos\alpha - |y_1|\right) \end{cases}$$

The explicit value of v in terms of u is not immediate, it is preferable to represent the backfill boundary by an approximate function :

$$u = m_b + a_b \cos v \quad (17)$$

The equations system is not linear as a function of α, but is solved from special points for which the exact value is known ($v=0$ and $v=-\pi$) :

$$m_b = \frac{1}{2}\ln\left[\frac{\left(L_b^{'} + r_b - |y_1|\right)\left(-L_b^{'} + r_b + |y_1|\right)}{\left(L_b^{'} + r_b + |y_1|\right)\left(L_b^{'} - r_b + |y_1|\right)}\right] \quad (18)$$

$$a_b = \frac{1}{2}\ln\left[\frac{\left(L_b^{'} + r_b - |y_1|\right)\left(L_b^{'} - r_b + |y_1|\right)}{\left(L_b^{'} + r_b + |y_1|\right)\left(-L_b^{'} + r_b + |y_1|\right)}\right] \quad (19)$$

For standard cables configurations, the error on u due to the approximation is in the order of 5%.

Boundaries are not straight horizontal lines in the transformed plane, and heat fluxes are not straight up. But we do a new assumption to evaluate the thermal resistances, as suggested in the solution by manual method of [4].

- hyp.6 : heat flow in the transformed plane is in paths parallel to the u axis (straight vertical lines).

The value of an elementary vertical resistance is :

$$\int_{u_e}^{0} \rho\, du = -\rho_b u_e + \left(\rho_b - \rho_e\right)\left(m_b + a_b \cos v\right) \quad (20)$$

The self thermal resistance R_{ii} of the cable i is given by equ.7 and 21 :

$$\frac{1}{R_{ii}} = \int_{-2\pi}^{0} \frac{1}{\int_{u_e}^{0} \rho du} dv \quad (21)$$

$$R_{ii} = \frac{1}{2\pi}\sqrt{\left(\rho_b \ln\left(u + \sqrt{u^2-1}\right) - \left(\rho_e - \rho_b\right)m_b\right)^2 - \left(\rho_e - \rho_b\right)^2 a_b^2} \quad (22)$$

The mutual thermal resistance R_{ik} between cable i and cable k is the resistance between the isothermal line issued from the centre of the cable k (co-ordinates z_k), and the ground surface.

$$v_k = \Im\left(\Phi(z_k)\right) = \arg\left[\frac{z_k + j|y_1|}{z_k - j|y_1|}\right] \quad (23)$$

According to hyp.6, isothermal lines may be considered as straight horizontal lines :

$$R_{ik} = \frac{1}{2\pi}\int_{u_k}^{0} \rho du \quad (24)$$

$$R_{ik} = \frac{1}{2\pi}\left[\rho_b \ln\frac{d_{ik}^{'}}{d_{ik}} - \left(\rho_e - \rho_b\right)\left(m_b + a_b \cos v_k\right)\right] \quad (25)$$

If the n cables of the system have approximately equal losses, the overall effective thermal resistance can be calculated for the cable of interest i :

$$R_i = T_4(i) \approx \sum_{k=1}^{k=n} R_{ik} \quad (26)$$

9. A comparison of the different methods

The iterative solution with conformal transform of [4] is reliable with a good accuracy. Complex designs with several circuits and different values of thermal resistivity can be computed. However the calculation time may be in a matter of minutes, and the method is not suitable in a design aid software based on analytical formulae (IEC 60287 or Neher-McGrath). Finite-difference or finite-element method tools may be more appropriate.

The straightforward Neher-McGrath method gives immediate results but the approximation of a rectangular backfill by an isothermal circle is only valid for a limited range of height/width ratios. The extended values of geometric factor of [6] are available in a table and overcome this restriction.

The direct method we have developed (with a non isothermal backfill) follows the variations of the reference method of conformal transform with a good accuracy, but needs to be rectified by a correcting factor (k_b = 1,017).

From the reference example without superficial layer, computations of the external thermal resistance have been performed for the different methods. Only one parameter is changed for a range of values, all other things being equal. For the hottest cable, the relative error (%) against the conformal transform result is then plotted on the following diagrams :

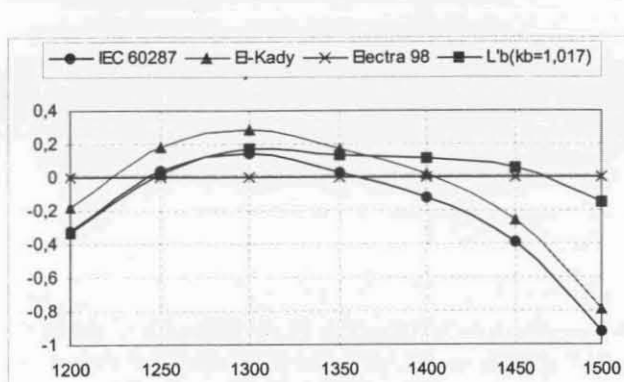

fig.3 : Variation of the cable laying depth

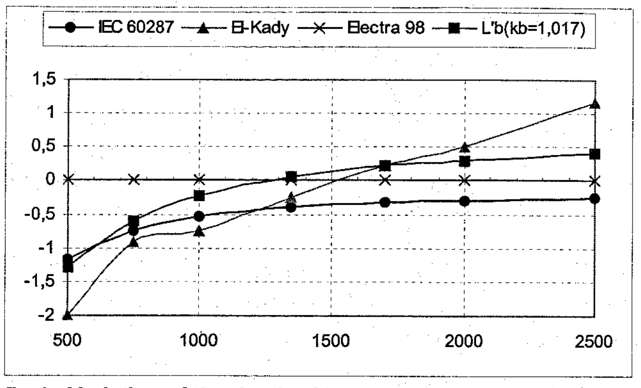

fig.4 : Variation of the depth of laying to centre of the backfill

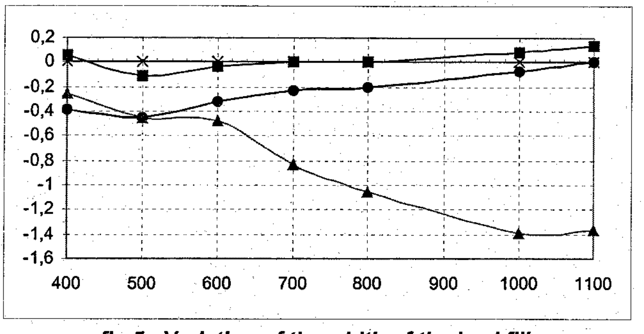

fig.5 : Variation of the width of the backfill

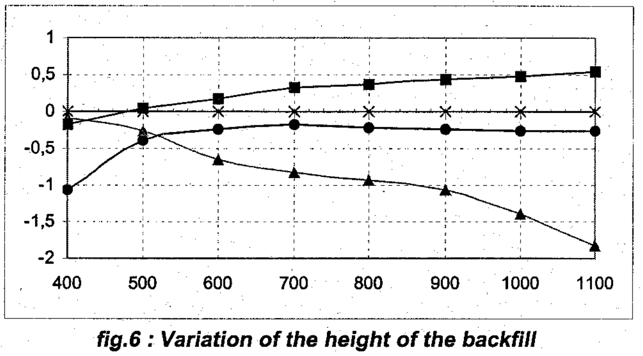

fig.6 : Variation of the height of the backfill

10. A simple method for a superficial layer

In the previous sections, the backfill of fig.1 is the only medium of different thermal resistivity considered. Cables are now assumed to be directly buried in the soil, with a superficial layer of height H_{sl} and thermal resistivity ρ_{sl}.

Let us calculate the conformal transform of the layer boundary :

$$w = u + jv = \Phi(x - jH_{sl}) \qquad (27)$$

where : $x \in \left]-\infty, +\infty\right[$,

According to equ.1, developing the exponential terms of equ.27, and identifying real and imaginary parts :

$$\begin{cases} e^u x.\cos v + e^u(|y_1| + H_{sl})\sin v = x \\ e^u x.\sin v - e^u(|y_1| + H_{sl})\cos v = |y_1| - H_{sl} \end{cases} \qquad (28)$$

Eliminating x, the function between u and v is :

$$(|y_1 + H_{sl}|)e^{u} - (|y_1 - H_{sl}|)e^{-u} = 2H_{sl}\cos v \qquad (29)$$

The explicit value of v in terms of u is not immediate, it is preferable to represent the superficial layer boundary by an approximate function :

$$u = m_{sl} + a_{sl}\cos v \qquad (30)$$

Substituting this expression in equ.28, using limiting forms, and assuming H_{sl} much less than y_1 :

$$\left(e^{a_{sl}}\right)^{\cos v} \approx 1 + \frac{|y_1| - H_{sl}}{|y_1| + H_{sl}}\left(e^{a_{sl}} - 1\right)\cos v \qquad (31)$$

$$\left(e^{-a_{sl}}\right)^{\cos v} \approx 1 + \frac{|y_1| + H_{sl}}{|y_1| - H_{sl}}\left(e^{-a_{sl}} - 1\right)\cos v \qquad (32)$$

Equ.29 becomes :

$$\begin{aligned} &(|y_1 + H_{sl}|)\left(e^{m_{sl}} - e^{-a_{sl}}\cos v\right) \\ &- (|y_1 - H_{sl}|)\left(e^{-m_{sl}} - e^{a_{sl}}\cos v\right) = 0 \end{aligned} \qquad (33)$$

This equation must be checked for any value of v :

$$\begin{cases} m_{sl} = -\dfrac{1}{2}\ln\left(\dfrac{|y_1| + H_{sl}}{|y_1| - H_{sl}}\right) \\ a_{sl} = \dfrac{1}{2}\ln\left(\dfrac{|y_1| + H_{sl}}{|y_1| - H_{sl}}\right) \end{cases} \qquad (34)$$

Since $H_{sl} \ll y_1$, then $a_{sl} \ll 1$, and the assumption relative to limiting forms (equ.31 & 32) is checked. The error on the boundary of the superficial layer in the transformed plane has been numerically estimated for a ratio H_{sl}/y_1 :

H_{sl}/y_1	error (%)
0,25	0,2
0,5	1
0,75	10

In any case, the superficial layer should not extend to the cables or the backfill, otherwise complex geometrical figures would be obtained in the transformed plane.

The self and mutual thermal resistances are given by similar expressions to equ.20, 21, 23 & 24 :

$$R_{ii} = \frac{1}{2\pi}\sqrt{\begin{aligned}&\left(\rho_e \ln\left(u + \sqrt{u^2 - 1}\right) - (\rho_{sl} - \rho_e)m_{sl}\right)^2 \\ &- (\rho_e - \rho_{sl})^2 a_{sl}^2\end{aligned}} \qquad (35)$$

$$R_{ik} = \frac{1}{2\pi}\left[\rho_e \ln\frac{d'_{ik}}{d_{ik}} - (\rho_{sl} - \rho_e)(m_{sl} + a_{sl}\cos v_k)\right] \qquad (36)$$

From the reference example without backfill, computations of the external thermal resistance have been performed for this simple method, changing the height of the superficial layer. For the hottest cable, the relative error (%) against the conformal transform result is then plotted on fig.7 :

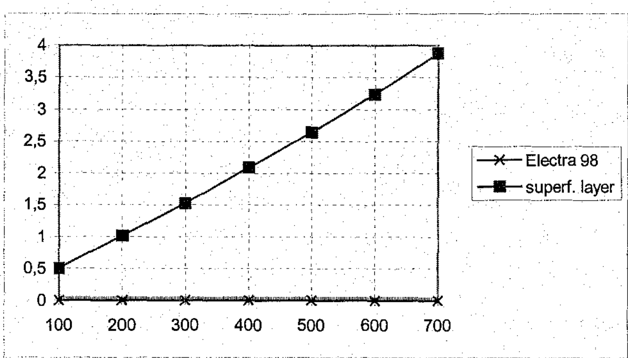

fig.7 : Variation of the height of the superficial layer

11. List of symbols

D_e = external diameter of cable (mm).
S = axial separation of conductors (mm).
L = depth of laying, to cable axis or centre of trefoil (mm).
L_G = distance from the soil surface to the centre of the backfill (mm).
W_b = width of the backfill (mm).
H_b = height of the backfill (mm).
H_{sl} = height of the superficial layer (mm).
ρ_e = thermal resistivity of earth surrounding the backfill (K.m/W).
ρ_b = thermal resistivity of material used for the backfill (K.m/W).
ρ_{sl} = thermal resistivity of the superficial layer (K.m/W).

y_1 = co-ordinate of line source of heat which corresponds to an isotherm at the surface of the reference cable (mm).
u_e = position of surface of reference cable in the transformed plane.
u,v = co-ordinates in the transformed plane.
R_{ii} = self thermal resistance of cable i (K.m/W).
R_{ik} = mutual thermal resistance between cable i and cable k (K.m/W).
R_i = sum of weighted thermal resistance of cable i (equivalent to T_4 in IEC Publication) (K.m/W).
z_k = complex co-ordinates of the centre of cable k in the original plane (mm).
v_k = co-ordinate on the horizontal axis of the centre of cable k in the transformed plane.

x,y = sides of backfills (x<y) (mm).
r_b = radius of the equivalent circular backfill (mm).
u,u_b = intermediate variables used in the calculation of the thermal resistance of surrounding medium of a cable, or in the calculation of the geometric factor of a rectangular backfill.
G_b = geometric factor of the backfill.
T_4 = thermal resistance of surrounding medium (K.m/W).
T_4' = thermal resistance of the air space between the cable surface and duct internal surface (K.m/W).
T_4'' = thermal resistance of the duct (K.m/W).
T_4''' = external thermal resistance of the duct (K.m/W).
T_{40}''' = original thermal resistance of surrounding medium before correction taking into account the backfill (K.m/W).
T_{4b} = thermal resistance between the cable surface or external surface of duct and the backfill boundary (K.m/W).
T_{4s} = thermal resistance between the backfill boundary and ground surface (K.m/W).
d_{ik} = distance from centre of reference cable i to an adjacent cable k (mm).
d'_{ik} = distance from centre of reference cable i to the image of an adjacent cable k (mm).

L_b = depth of the equivalent circular backfill (mm).
L_b' = corrected depth of the equivalent circular backfill (mm).
m_b,a_b= mean and amplitude values of the approximate function to represent the circular backfill boundary in the conformal transformed plane.
m_{sl},a_{sl}= mean and amplitude values of the approximate function to represent the superficial layer boundary in the conformal transformed plane.
k_b = factor to rectify the global thermal resistance for the corrected backfill depth method.

12. References

[1] International Standard IEC 60287-1-1, Electric cables - Calculation of the current rating, Part 1: Current rating equations (100% load factor) and calculation of losses, Section 1 : General, 1994.
[2] International Standard IEC 60287-2-1, Electric cables - Calculation of the current rating, Part 2 : Thermal resistances, Section 1 : Calculation of thermal resistance, 1994.
[3] International Standard IEC 60287-3-1, Electric cables - Calculation of the current rating, Part 3 : Sections on operating conditions, Section 1 : Reference operating conditions and selection of cable type, 1995.
[4] Working Group 02 (Cable Ratings) of Study Committee 21 (HV Insulated Cables), The calculation of the effective external resistance of cables laid in materials having different thermal resistivities, Electra n°98, January 1985.
[5] G.J. ANDERS, Rating of Electric Power Cables - Ampacities computations for transmission, distribution and industrial applications, 1997.
[6] M.A. EL-KADY, D.J. HORROCKS, Extended values for Geometric factor of external thermal resistance of cables in duct banks, IEEE Transactions on Power Apparatus and Systems, Vol. PAS-104, N°8, August 1985.
[7] G. MERMET, U. SPORREL, Underground cables in a backfill , Supélec industrial study report for SAGEM, March 1999.